Effects of isospin-symmetry violation on tests of the standard model using parity-violating electron scattering

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Abstract

The effects of isospin-symmetry breaking on the observables for parity-violating electron scattering are investigated within the framework of the nuclear shell model for ^{12}C, ^{16}O, and ^{28}Si. Contributions due to mixing with low-lying states as well as admixtures of $1p-1h$ configurations (via the radial wave functions) are accounted for. It is found that isospin-mixing can be important, and the consequences regarding precision tests of the standard model are addressed.

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One of the most successful theories in physics is the standard model for the electroweak interaction [1]. Because of its extraordinary success and the fact that the origin of many of the parameters defining it are not that well understood, the focus of many programs is to perform high-precision experiments with the hope of discovering fingerprints of physics beyond the standard model. Atomic nuclei provide a convenient laboratory for these tests, although in some cases nuclear-structure effects must be accounted for. Two examples are: (1) the ft values for superallowed Fermi beta decay [2–5], which test the conserved vector current hypothesis [6] and the unitarity of the Cabibbo-Kobayashi-Maskawa matrix [7]; and (2) parity violation in electron scattering from even-even, $N = Z$ nuclei, which offers a window into the neutral-current sector of the weak interaction [8–10]. In both examples is that the systems were chosen to minimize the effects due to the internal structure of the nucleus. In fact, if isospin is a good quantum number, the measured observables are hardly affected by nuclear structure at all. However, because of the presence of the Coulomb interaction and charge-dependent components in the strong interaction, isospin symmetry is violated and corrections can be expected. In the case of Fermi beta decay, the corrections are small ($\leq 0.7\%$), but important [2–5]. In the case of parity-violating electron scattering, the goal is to perform a measurement that provides a 1% test of the standard model. Effects due to isospin-symmetry breaking on the parity-violation observables have been estimated previously [11], with the conclusion being that for ^{12}C isospin-mixing corrections are expected to be less than 1% for essentially all momenta transfer.

With a series of experiments now planned for TJLAB, this issue is revisited using improved nuclear models. The principal improvements implemented are: (1) using radial wave functions obtained from Hartree-Fock (or Woods-Saxon) calculations with separation energies determined from intermediate states of the $A - 1$ system to account for the mixing between the ground state and one particle-one hole $(1p - 1h)$ excitations and (2) performing the shell-model calculations in proton-neutron formalism while including empirically determined isospin-nonconserving (INC) interactions [12]. The advantages in (2) are that the interaction inducing isospin mixing is constrained to reproduce binding energy differences

within isospin multiplets and that the sum over states that can mix into the ground state is carried out implicitly. For (1), an important action of the Coulomb force is to introduce admixtures of $1p - 1h$ states that effectively renormalize the proton radial wave functions relative to the neutrons by decreasing the single-particle separation energy. Although these effects are in principal included in the definition of the Hartree-Fock mean field, it is important to correctly account for the separation energy of the single-particle states relative to the complete set of states in the $A - 1$ system. These procedures are in contrast to those of Ref. [11], where a more qualitative result was obtained primarily through a two-level model and adopting a "worst-case" philosophy assuming isospin-mixing matrix elements of the order 300 keV.

The observable of interest for elastic scattering from an even-even, $N = Z$ target is the parity-violating electron scattering asymmetry [11,13]

$$\mathcal{A} = \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-} = -\left(\frac{G_F q^2}{4\pi\alpha\sqrt{2}}\right)\frac{\tilde{F}_C(q)}{F_C(q)}, \tag{1}$$

where $\pm$ refers to electrons with helicity ± 1, G_F is the weak interaction Fermi constant, α is the fine structure constant, and $q = |\mathbf{q}|$ is the magnitude of the three momentum transfer. The dependence on nuclear structure is embodied in the Coulomb form factors $F_C(q)$ and $\tilde{F}_C(q)$ for the electromagnetic and neutral currents, respectively. Both form factors have the same form, with the primary difference only being in the charges. In particular, the electromagnetic form factor is given by

$$F_C(q) = \sum_{\mu}^{protons} n_\mu^p f_\mu^p(q) + \sum_{\mu}^{neutrons} n_\mu^n f_\mu^n(q), \tag{2}$$

where μ denotes the labels for each single-particle orbit, $n_\mu^{p(n)}$ the number of protons(neutrons) occupying each single-particle orbit, and $f_\mu^{p(n)}(q)$ is given by

$$f_\mu^{p(n)}(q) = \int_0^\infty r^2 dr (R_\mu^{p(n)}(r))^2 \hat{M}_0^{p(n)}(qr), \tag{3}$$

where $R_\mu^{p(n)}(r)$ is the radial wave function, and $\hat{M}_0^{p(n)}(qr)$ is a generalized charge operator given by

$$\hat{M}_0^{p(n)}(qr) = \frac{1}{\sqrt{4\pi}} \left[\frac{G_E^{p(n)}(\tau)}{\sqrt{1+\tau}} j_0(qr) + (G_E^{p(n)}(\tau) - 2G_M^{p(n)}(\tau)) \frac{2\tau j_1(qr)}{qr} \sigma \cdot \mathbf{1} \right] \tag{4}$$

where $\tau = q^2/4m_N^2$, m_N is the mass of the nucleon, and $G_E^{p(n)}$ and $G_M^{p(n)}$ are the Sachs [14] electric and magnetic intrinsic form factors. Here, the dipole forms of Ref. [15] are used. For the neutral current, $\tilde{F}_C(q)$ is also given by Eqs. (2)-(4) using the intrinsic weak form factors [9]

$$G_{E(M),W}^p = [(1 - 4\sin^2\theta_W)G_{E(M)}^p - G_{E(M)}^n]$$
$$G_{E(M),W}^n = [-G_{E(M)}^p + (1 - 4\sin^2\theta_W)G_{E(M)}^n], \tag{5}$$

with $\sin^2\theta_W \sim 0.230 \pm 0.005$ [16]. Lastly, a correction due to the center-of-mass must also be applied. For a harmonic oscillator potential, this is well defined [17], and Eq. (2) is multiplied by $\exp(a_0^2 q^2/4A)$, where $a_0^2 = m_N\omega/\hbar$ is the oscillator parameter, which may be accurately parameterized with $\hbar\omega = 45A^{-1/3} - 25A^{-2/3}$ MeV. Although this simple decomposition is not possible for the more realistic potentials used in Woods-Saxon or Hartree-Fock calculations, the harmonic-oscillator result represents a reasonable approximation.

In the limit that isospin is a good quantum number, $n_\mu^p = n_\mu^n$ and $R_\mu^p(r) = R_\mu^n(r)$, and Eq. (1) reduces to

$$\mathcal{A}_0 = [G_F q^2/\pi\alpha\sqrt{2}] \sin^2\theta_W = 3.22 \times 10^{-6} q^2, \tag{6}$$

with q^2 measured in fm^{-2}. It is this simple form that makes experiments on even-even, $N = Z$ nuclei an attractive choice for testing the standard model, as any deviation from the simple q^2 dependence might be a signature of physics beyond the standard model. On the other hand, isospin is not a conserved quantity and corrections to Eq. (6) must be accounted for. These corrections may be embodied in the factor $\Gamma(q)$ [11] defined as

$$\mathcal{A} = \mathcal{A}_0(1 + \Gamma(q)), \tag{7}$$

which, from Eq. (1), may be written as

$$\Gamma(q) = -[1 + \tilde{F}_C(q)/2\sin^2\theta_W F_C(q)]. \tag{8}$$

An estimation of $\Gamma(q)$ begins by noting that as in the case of superallowed Fermi beta decay, two types of isospin mixing must be accounted for [3]. The first contribution is due to mixing between states contained within the shell-model configuration space. For example, the ^{12}C model space consists of the $0p_{3/2}$ and $0p_{1/2}$ orbitals (p-shell), and there are 5, 2, and 2 configurations leading to $J^\pi = 0^+$ and $T = 0, 1$, and 2, respectively. The INC interaction is composed of isospin operators of rank zero, one, and two, and consequently mixes together all the $J^\pi = 0^+$ states in ^{12}C. In regards to Eq. (2), isospin mixing within the configuration space leads to $n_\mu^p \neq n_\mu^n$.

In addition to the mixing between states within the configuration space, mixing with states that lie outside the model space must also be taken into account. The Coulomb interaction can strongly mix $1p - 1h$, $2\hbar\Omega$ excitations, eg., $0p_{3/2} \to 1p_{3/2}$, into the ground state. For Fermi transitions, these excitations were accounted for by examining differences in the proton and neutron radial wave functions [3–5]. For closed-shell configurations, the mixing between the ground state and $1p - 1h$ states is contained within the Hartree-Fock (HF) procedure, and an estimate of this contribution might be obtained by evaluating the $f_\mu^{p(n)}$ using HF radial wave functions.

As a start, ^{12}C is examined in detail. The occupation factors $n_\mu^{p(n)}$ were obtained from shell-model calculations carried out in proton-neutron formalism within the p-shell. In addition to the isospin-conserving shell-model Hamiltonian CKPOT [18], isospin mixing within the configuration space was accounted for by including the INC Hamiltonian of Ref. [12]. The INC interaction contains both Coulomb and charge-dependent nucleon-nucleon terms whose strengths were determined by the requirement that binding energy differences within isospin multiplets be reproduced. Interactions of this form have subsequently been used to examine isospin-mixing corrections to Fermi beta decay [5], isospin-forbidden particle emission [19], isospin-forbidden Fermi beta decay [20], and to predict the location of the proton-drip line [21]. For the $f_\mu(q)$, radial wave functions obtained from HF calculations utilizing Skyrme-type interactions [22] and the shell-model occupation factors to define the HF densities were used. Here, the M* [23] interaction was used, but others produced quali-

tatively the same results. In Fig. 1a, the square of the charge form factor obtained with HF radial wave functions (dotted line) is compared with experimental data [25]. The HF result reproduces the experimental data up to the diffraction minimum, but is in serious disagreement beyond. This region is quite sensitive to the details of the radial wave functions, and in particular the separation energy, which can be obtained from the experimental binding energies for ^{11}B and ^{11}Cs [24]. Towards this end, the same procedure used to evaluate the radial-overlap correction for Fermi beta decay [3,4] may be used. The occupation factors may be rewritten as

$$n_\mu = \sum_\pi \tfrac{1}{2} S(\mu, \pi),$$

(9)

where the sum extends over the complete set of states $|\Psi(\pi)\rangle$ of the intermediate $A - 1$ system, and the spectroscopic factor $S(\mu, \pi)$ is given by

$$S(\mu, \pi) = \frac{|\langle \Psi(^{12}C)|a_\mu^\dagger|\Psi(\pi)\rangle|^2}{2J_\pi + 1}.$$

(10)

The f_μ are then evaluated for each intermediate state by scaling the central part of the mean-field potential to yield the correct separation energy between the ^{12}C ground state and the intermediate state $|\Psi(\pi)\rangle$ [24]. The corresponding charge form factor is illustrated by the solid line in Fig. 1a, where it seen that much better agreement with experiment is achieved.

Shown in Fig. 1b is the expected parity-violating asymmetry (solid line) compared with the "pure" standard model expectation (dashed line). In Fig. 1c, $|\Gamma(q)|$ is plotted (solid line), and is seen to increase rapidly; exceeding the critical value of 1% at approximately 0.9 fm^{-1}. The large increase in $\Gamma(q)$ is due to the Coulomb interaction "pushing" the proton distribution out relative to the neutrons, leading to a shift in the diffraction minima between the Coulomb and weak form factors. At the diffraction minimum, $\Gamma(q)$ changes sign, and actually crosses through zero again in a relatively flat region near the second maximum. This is a general feature that might be exploited for experiments on heavier nuclei.

Also shown in Fig. 1c, is the relative dominance of the contribution to $\Gamma(q)$ due to the radial wave function. The dashed line represents $\Gamma(q)$ evaluated using isospin-conserved

occupation factors in conjunction with proton and neutron radial wave functions, while the dotted line shows the contribution due to isospin mixing within the shell-model space (begins negative), and was evaluated by replacing $f_\mu^p(q)$ with $f_\mu^n(q)$.

$\Gamma(q)$ was also evaluated for ^{16}O and ^{28}Si. For ^{16}O, the closed p-shell was assumed, while for ^{28}Si, shell-model calculations were carried out in proton-neutron formalism within the sd-shell ($0d_{5/2}$, $0d_{3/2}$, and $1s_{1/2}$ orbitals) model space using the isospin-conserving USD Hamiltonian of Wildenthal [26] and the sd-shell INC interaction in Ref. [12]. For ^{16}O and ^{28}Si, the effects of the intermediate $A - 1$ states were taken into account. The resulting values of $|\Gamma(q)|$ are shown in Fig. 2 and compared with ^{12}C. The 1% value is illustrated by the dot-dashed line.

Another nucleus of interest is ^{4}He. Because of the small charge, isospin-mixing corrections are expected to be small. For completeness, $\Gamma(q)$ was also evaluated for ^{4}He assuming a closed $0s_{1/2}$ core, and is illustrated in Fig. 2. Although $\Gamma(q)$ is found to be small, and in overall agreement with the results of Ref. [27], a weakness in the calculation is the position of the diffraction minimum, which illustrates the need to go beyond the closed $0s_{1/2}$ configuration and to include meson-exchange currents [28]. Another approach currently under investigation, is to perform a large-basis, no-core shell-model calculation utilizing a realistic interaction [29] that also includes INC components, as was recently done for the superallowed Fermi transition in ^{10}C [30]. In this case, both contributions to isospin mixing are contained within the same formalism. This calculation would be an excellent compliment to earlier works that utilized variational ^{4}He ground-state wave functions [28,27].

In addition to providing a test of the standard model, parity-violating electron scattering is also sensitive to any strangeness content in the nucleon [31], and can be used as a probe to measure the strangeness form factor. These experiments will be carried out at higher momenta transfer, and from Fig. 2 it is clear that they have to be carefully designed in order to minimize effects due to nuclear structure. Towards this end, it is likely that the cross over through zero exhibited in $\Gamma(q)$ near the second maximum of $F_C^2(q)$ may be exploited.

In conclusion, effects due to isospin-mixing on the observables for parity-violating elec-

tron scattering were evaluated for ^{12}C, ^{16}O, and ^{28}Si. The method employed very closely mirrors that used for superallowed Fermi beta decay. In particular, the influence of the intermediate parent states in the $A - 1$ system was found to be important for a proper description of the charge form factor. In general, it is found that $\Gamma(q)$ increases rapidly with q, and often exceeds the critical value of 1%. However, $\Gamma(q)$ is also found to "cross-over" and pass through zero in a region just past the second maximum in the charge form factor. It might be possible to exploit this feature to make experiments on heavier nuclei amenable to tests of the weak interaction or as a probe of the strangeness content in the nucleon.

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FIGURES

FIG. 1. ^{12}C results. In (a), $F_C^2(q)$ is plotted and compared with experimental data. The dotted line represents the results obtained with the Hartree-Fock radial wave functions, while the solid line respresents the results obtained by summing over the intermediate $A-1$ states as explained in the text. In (b) and (c) the parity-violation assymmetry parameter, $\mathcal{A}$, and isospin-mixing correction $|\Gamma(q)|$ (in %, solid line), respectively, are plotted. In (c), the contributions to $\Gamma(q)$ due to the differences in the radial wave functions and isospin mixing within the shell-model space are illustrated by the dashed and dotted lines, respectively.

FIG. 2. Calculated values of $|\Gamma(q)|$ (in %) obtained for ^{4}He (long dashed), ^{12}C (solid), ^{16}O (dotted), and ^{28}Si (dashed).